Tracking Control of a Piezo Actuated System using Fuzzy Sliding-Mode Control with Feedforward Compensation

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Abstract

In this paper, simple fuzzy sliding mode control is used to resolve tracking problem of a piezo-actuated stage. The piezo-actuated system is composed of the piezoelectric actuator (PEA) and a positioning mechanism. Due to hysteretic nonlinearity of the PEA, the tracking accuracy of the system is limited. To compensate for this nonlinearity, a feedback fuzzy sliding-mode control (F-FSMC) augmented with a predictor-based feedforward compensator is proposed. The controller, denoted as the feedforward-feedback fuzzy sliding-mode controller (FF-FSMC), can be applied to eliminate tracking error caused by the hysteretic characteristics using a minimal knowledge of the system. Experimental results on several types of reference inputs indicate that the proposed control schemes may suppress the tracking error to within 1.4% full span range (FSR) of the actuator in 1Hz and 5Hz sinusoidal waveform input, and to within 4% FSR in a 10-Hz band-limited random signal tracking.

1. Introduction

Piezoelectric actuators (PEAs) can be electrically controlled to move with a resolution on the order of 1 nm. This characteristics, together with its fast response, high stiffness, no backlash, small size advantages render the PEA an essential component in ultra-precision machines such as scanning tunneling microscopes and diamond turning machine. Piezoelectric actuators also exhibit undesired hysteretic behaviors. The hysteretic behaviors limit performance of the PEA-actuated system. As such, recently, many control techniques involving feedback and feedforward-feedback features have been proposed to eliminate the undesirable hysteresis effects [15,17-20]. Among these attempts, it is found that usually, the feedback control techniques do not utilize a precise hysteretic model. On the other hand, the feedforward-feedback algorithms deal with the hysteresis and structural dynamics separately, i.e., apply a compensator in the feedforward loop to cancel the undesired effects caused by the hysteresis, and then design a feedback controller to control the structural dynamic effect. The feedforward compensator may cancel out nonlinear hysteretic effects; however, the effectiveness of the method strongly relies on the accuracy of the system model.

In order to save tremendous modeling efforts that are frequently required in the model-based control methods, two general types of control schemes, including the fuzzy logic control (FLC) and the sliding mode control (SMC), are employed to seek out the success of intelligent control schemes. Thereby, a feedback fuzzy sliding-mode controller (F-FSMC) is proposed in this study to control the structural dynamics of the piezo-actuated system. The proposed schemes first adopt pre-defined sliding function and its first derivative as the input variables for the FLC to generate the ensuing fuzzy logics. The controller then mimics the approach in the traditional sliding mode control. In doing so, advantages associated with the FLC and SMC can be achieved in the proposed schemes while their drawbacks can be avoided.

The idea of using FLC to complement SMC or vice versa is not new. In fact, the F-FSMC has been adopted in [16] to perform the position control of a ball screw driven by pneumatic servomotor. Experimental results in [16] show good performance of the F-FSMC. In addition, [5], [7, 2] separately proposed to control a motor-mechanism system and an axially moving string via the F-FSMC. Dynamic behavior of the proposed controller-motor-mechanism systems in [5] has been shown to be robust with respect to external disturbances, whereas the transverse vibration in [7, 2] can be well suppressed via the application of the F-FSMC. The F-FSMC was also adopted in [1] to control structures in which the chattering effect, the major disadvantage of conventional sliding-mode controller, was removed while the robustness against parameter uncertainties, modeling inaccuracies and time-varying dynamics is kept.

In addition to the feedback fuzzy sliding-mode controller, a predictor-based feedforward compensator is proposed and integrated with the F-FSMC in this paper. Unlike other feedforward compensators described previously, which require precise knowledge of the hysteresis model so as to cancel the undesired hysteretic effects, a first order on-line feedforward compensation is adopted here. The compensation method adopted is more like a direct compensation rather than a characteristic-based one for the predicted tracking error obtained based on the knowledge of previous tracking errors is directly applied to compensate the real tracking error. Similar compensation idea has been applied in [9,13] where effective control performances were
achieved. Specifically, a quadratic polynomial predictor model was adopted in [9] to control a piezo-actuated tool servo of a diamond turning machine. It was found that satisfactory tracking performance could be obtained by using the feedback PI controller integrated with the feed-forward predictor. On the other hand, it has been shown in [13] that by iteratively feedforwarding predicted error to the control system, i.e. N-order instead of first-order compensation were used, the real tracking error would always be decreasing as the order of compensation increased. Generally, the prediction is based on the knowledge of the previous tracking errors (or state) together with an assumed polynomial model. In this paper, a first-order compensator equipped with a quadratic polynomial predictor serves as a complement to the F-FMSC. Since the design of the F-FSMC depends to a large extent on the expert’s knowledge or on trial and error, integrating the first-order compensator with the F-FSMC can substantially save the tuning effort.

2. The Piezo-Actuated System and Its Open-Loop Behaviors

2.1 The Experimental Set-Up

Our experimental set-up composes of a mechanical stage supported by two flexible beams, a piezoelectric stack, serving as the actuator, and a displacement sensor (which in this study is a subminiature load cell). The piezoelectric actuator studied in this paper is Model PST500/25 of Piezomechanik GmbH, which has a maximum stroke of 25µm corresponding to an input voltage around 500 volt. A power amplifier (Model PosiCon.an 150-3, Piezomechanik GmbH) with a linear gain of 30 is used to drive the piezoelectric actuator and a flexible stage while a strain-gauge type subminiature load cell (Honeywell, Model 13) is mounted to measure the reaction force acting against the stack and the stage. The force measurement was then converted to the stage displacement through a calibration relationship. The calibration was accomplished by using a non-contact Doppler laser-beam displacement sensor (SIOS MI-5000). Figure 1 presents the schematic diagram of the fuzzy control system.

The tracking control was implemented using a personal computer augmented with two plug-in control boards. Both A/D and D/A functions are provided in these control boards (Advantech PCL-812PG and PCL-726) so that the digital data acquisition and real-time control tasks can be performed simultaneously. Figure 1 shows that a signal-conditioning device, consisting of Wheatstone bridge circuit, conditioned the force measurements while the voltage output from PC was amplified using a high-voltage power amplifier before fed into the piezoelectric actuator. The output of the voltage amplifier is used to manipulate the piezoelectric actuator. When the driving voltage of 500V is applied, the maximum displacement of the PEA without loading is around 25µm. Meanwhile, results of system identification indicate that the stiffness of the piezo-actuated system as a whole is about 13,450,574N/m while the mass of the system is about 0.75kg. These values result in a fundamental natural frequency equal to 4234rad/sec or 674Hz. Due to the high stiffness of the system, the voltage output of the force sensor could be larger than its capacity when the displacement of the PEA is large. To that end, the maximum displacement allowed in our experimental set-up is about 2µm.

2.2 Open-Loop Behaviors of the System

In order to demonstrate the nonlinear hysteretic characteristics of the piezo-actuated system, a couple of open-loop tests were carried out, and the results are presented in Figure 2. Figure 3 illustrates hysteretic loop structures appearing in the input-output relationship of the piezo-actuated system during an open-loop test. Obviously, the nonlinear hysteretic characteristics exist in our experimental system. In Figure 2, the input signal was a triangular waveform with decreasing positive span. Therefore, the hysteresis loops converge toward the left-bottom corner. If the operating voltage decays at the midpoint of the input span, the hysteretic loops will converge toward the center in Figure 2. It can also be seen in Figure 2 that the response of the piezoelectric actuator shows a constant residual displacement near zero input voltage. Both the loop structure and the residual displacement register evident nonlinear hysteresis of the system. In addition, random noise were picked up from the environment and superimposed on the top of the system response. It is conceivable that the random noise has nothing to do with the dynamic of the system because its presence can be observed even though the system is not excited.

3. The Feedforward–Feedback Fuzzy Sliding-Mode Control

3.1 Design of the Feedback Fuzzy Sliding-Mode Controller (F-FSMC)

The control performance of a traditional controller, such as the sliding mode control, usually depends on the knowledge and accuracy of the assumed system dynamic model. Since the piezo-actuated system has nonlinear and time-varying behavior, it is difficult to derive a universally accurate model or estimate the bound of uncertainties for the traditional controller design. To that end, a model-free intelligent controller collaborating fuzzy logic control (FLC) with sliding mode control (SMC) is adopted in this study. Figure 5 shows the block diagram of the proposed feedforward-feedback fuzzy sliding-mode control (FF-FSMC).

In designing the FF-FSMC shown in Fig. 3, the sliding surface is defined using the following scalar function

\[ s = C_1 e + \dot{e} \]  

(1)

where \( e = x_d - x \) represents the tracking error of the stage displacement and \( C_1 \) is a strictly positive constant.
The sliding surface depicted in Eq. (1) represents a straight line in the phase plane. In practical situations, \( \dot{s} \) can be approximated with the following discrete form

\[
\dot{s}(KT) = \frac{1}{T}[s(KT) - s((K-1)T)]
\]  
(2)

where \( K \) denotes the number of sampling iteration and \( T \) is the sampling period. In order to accommodate various system characteristics for a better control performance, the input functions for the FSMC, \( s \) and \( \dot{s} \), are multiplied by scaling factors \( G_s \) and \( G_{\dot{s}} \), respectively. These scaling factors are adopted to equip the designers with the capability of adjusting control gains before \( s \) and \( \dot{s} \) were taken as input variables of the FLC. The scaling yields

\[
S = sG_s, \quad \dot{S} = \dot{s}G_{\dot{s}}
\]  
(3)

where \( S \) and \( \dot{S} \) are also known as the universe of discourse of \( s \) and \( \dot{s} \). In addition, since the output of the FSMC, \( \Delta U \), is in its corresponding universe of discourse of the change of the controller output, the actual output value, \( \Delta u \), can be obtained by introducing a scaling factor \( G_u \), namely

\[
\Delta u = \Delta U G_u
\]  
(4)

Then, the actual input sending to the piezoelectric actuator (after passing the notch filter) can be represented in the following discrete form

\[
u(KT + T) = u(KT) + \Delta u(KT + T)
\]  
(5)

According to the definitions described above, the fuzzy sets associate with these input and output variables can be determined as the followings:

- \( NB \): negative big,
- \( NM \): negative medium,
- \( NS \): negative small,
- \( ZR \): zero,
- \( PS \): positive small,
- \( PM \): positive medium,
- \( PB \): positive big.

In this study, the triangular membership function was used to fuzzify the input and output variables of the FLC into linguistic variables. Figure 4(a), (b), and (c) show the membership functions of \( S \), \( \dot{S} \), and \( \Delta U \), respectively. It can be seen from these figures that seven subsets, \( NB, NM, NS, ZR, PS, PM, PB \), are defined for \( S \) and \( \dot{S} \). As a consequence, 49 fuzzy rules are required to accomplish the fuzzy control design. The resulting fuzzy sliding-mode inference rules are presented in look-up tables as shown in Table 1, where \( \Delta U \) is the fuzzy mapped function of \( S \) and \( \dot{S} \). Note that the essence of rules in Table 1 is to satisfy the requirement of the Lyapunov stability theorem, i.e., \( SS < 0 \). An example of a rule thinking process can be explained as follows:

“If \( S \) and \( \dot{S} \) are negative big (NB), then \( \Delta U \) is negative big (NB).”

Finally, the fuzzy control variables are transformed to the form of the crisp numbers by using the height method in the module of defuzzification, leading to

\[
\Delta U = \frac{\sum_{i=1}^{m} \Delta U_i \mu_i}{\sum_{i=1}^{m} \mu_i}
\]  
(6)

where \( m \) indicates the number of rules, \( \Delta U_i \) denotes the fuzzified value of the change of the controller output and \( \mu_i \) is the weight of the corresponding rule which has been activated [7]. Note also that a notch filter is added in the forward loop to reduce the effects of structural resonant dynamics.

3.2 The Predictor-Based Feedforward Compensation

In order to eliminate the tracking error caused by the hysteresis, a simple predictor was added to the feed-forward loop of the fuzzy control system. According to the context of the \( N \)-order feedforward compensation defined in [13], the compensator adopted in this study can be viewed as an on-line first-order one, because only a single compensation action is taken. The predictor is implemented using the following quadratic polynomial prediction equation

\[
e(t) = at^2 + bt + c
\]  
(7)

where \( e(t) \) represents the tracking error while \( a, b, c \) are constant coefficients to be estimated in real time.

Since the errors measured at the past time are already known, these quantities can be applied to compute the coefficients of Eq. (7). Therefore, in discrete form, the following three algebraic equations are obtained

\[
e(t - T) = a(t - T)^2 + b(t - T) + c
\]  
(8a)

\[
e(t - 2T) = a(t - 2T)^2 + b(t - 2T) + c
\]  
(8b)

\[
e(t - 3T) = a(t - 3T)^2 + b(t - 3T) + c
\]  
(8c)

where \( T \) denotes the sampling period. By substituting coefficients \( a, b, c \) into the polynomials shown in Eq. (7), one is able to on-line predict the tracking error \( \hat{e}(t) \). The predicted value, denoted as \( \hat{e}(t) \), will be directly fed in the feedforward loop.

4. Experimental Investigations

To verify the proposed algorithm, two types of controllers are adopted and their results are compared. First, a feedback fuzzy sliding-mode controller (F-FSMC) without the predictor-based compensator was utilized. The merit of this controller is that it is simple in structure, model-free and easy to design. Nevertheless, in order to acquire the best performance, control action can only be accomplished by careful tuning the gains of the F-FSMC. Therefore, a lot of trial and error tuning efforts are required. In contrast with the F-FSMC, the feedforward-feedback fuzzy sliding-mode controller (FF-FSMC) proposed requires much less tuning efforts due to the presence of the compensator.

Experiments on tracking of sinusoidal waveform...
inputs at frequencies of 1 Hz and 5 Hz and magnitude of 0-1 μm and 0-2 μm are conducted to explore the performances of the proposed control scheme. To show the effectiveness of the proposed controller, Figures 5 presents the input-output relationships of the open-loop and closed-loop system where the closed-loop system refers to the system using the FF-FSMC. In Figure 5, the system was subjected to a sinusoidal waveform reference input of 1 Hz. It can be seen from this figure that hysteresis effects including the hysteretic loop and residual displacement were significantly reduced. Thus, the results indicate that the FF-FSMC can effectively reduce the nonlinear hysteretic effects introduced by the PEA.

Next, Figure 6 and 7 show, in time domain, the desired sinusoidal input versus the closed-loop displacement response using the F-FSMC and FF-FSMC, respectively. The associated tracking errors of each case are also presented and summarized in Table 2. The results presented in Figure 6 and 7 correspond to a 1 Hz sinusoidal waveform reference input. According to the results presented in Figure 6 and 7 one can find out easily that the FF-FSMC has better steady-state tracking performances than the F-FSMC. In fact, the maximum errors of the F-FSMC for reference inputs of 1 Hz and 5 Hz were 0.025 μm and 0.13 μm, respectively, whereas the errors of the FF-FSMC corresponding to the same inputs were 0.004 μm and 0.014 μm. Thus, the control accuracy of the FF-FSMC obviously outclasses that of the F-FSMC. Considering the fact that the F-FSMC requires a lot more tuning efforts than the FF-FSMC, the latter seems to be overwhelming.

Since in most of the practical applications the desired reference input will not be a pure sinusoidal waveform, tracking control of a band-limited random signal might be more suitable in examining the performance of the proposed method in the real world situation. To this end, Figures 8 and 9 show the experimental tracking results of a band-limited random input with a bandwidth of 10 Hz. In Figure 8, the results of the F-FSMC are presented while Figure 9 shows the results obtained from the FF-FSMC. Evidently, the accuracy of the FF-FSMC is again much better than that of the F-FSMC. According to the results listed in Table 2, the maximum tracking error of the F-FSMC for the random input is 17% of the FSR while those of the FF-FSMC is below 4%.

5. Conclusions

In this study, the inherent hysteresis of a piezo-actuated system has been demonstrated experimentally. The hysteresis makes accurate open-loop tracking control of the system impossible and complicates closed-loop control. A new tracking controller that combines a feedforward compensator with a feedback fuzzy sliding-mode control is presented. The merits of the proposed controller are two-folded. Firstly, the feedback fuzzy sliding-mode control is model-free while the predictor-based compensator adopted does not rely on model cancellation. Therefore, modeling on structure dynamic and nonlinear hysteresis is not necessary. Secondly, the predictor-based compensator can be used to complement the fuzzy sliding-mode control, because trial and error tuning efforts can be significant saved.

Experimental results show that tracking errors of the piezoe-actuated system can be significantly reduced using the proposed FF-FSMC. It is also found that the proposed control schemes can suppress the tracking error to within 1.4 % of the FSR of the piezoelectric actuator in 1 Hz and 5 Hz sinusoidal waveform input, and to within 4% FSR in a 10-Hz band-limited random signal tracking.

6. Acknowledgment

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7. References


### 8. Figures and Tables

#### Table 1 Linguistic rules based on FSMC for the piezo-actuated system

<table>
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<tr>
<th>$\Delta U$</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZR</th>
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<td>NS</td>
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<td>ZR</td>
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<tr>
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<td>NS</td>
<td>NS</td>
<td>ZR</td>
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<tr>
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#### Table 2 Tracking errors of sinusoidal and band-limited random input (displacement 0 to 1 $\mu m$)

<table>
<thead>
<tr>
<th>Tracking Error</th>
<th>F-FSMC</th>
<th>FF-FSMC</th>
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<tr>
<td>$\mu m$</td>
<td>%</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>1 Hz sinusoidal</td>
<td>0.025</td>
<td>2.5</td>
</tr>
<tr>
<td>5 Hz sinusoidal</td>
<td>0.130</td>
<td>13</td>
</tr>
<tr>
<td>10Hz band-limited random signal</td>
<td>0.170</td>
<td>17</td>
</tr>
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</table>
Figure 5 Open-loop and closed-loop input-output relationships of a 1-Hz sinusoidal waveform reference input.

Figure 6 (a) The comparison between the desired 1-Hz sinusoidal input (solid line) and displacement response (dashed line) of a closed-loop test using the F-FSMC (b) the tracking error of (a).

Figure 7 (a) Comparison between the desired 1-Hz sinusoidal input (solid line) and the displacement response (dashed line) of a closed-loop test using the FF-FSMC (b) the tracking error of (a).

Figure 8 (a) Comparison between the desired 10-Hz band-limited random input (solid line) and displacement response (dashed line) of a closed-loop test using the F-FSMC (b) the tracking error of (a).

Figure 9 (a) Comparison between the desired 10-Hz band-limited random input (solid line) and displacement response (dashed line) of a closed-loop test using the FF-FSMC (b) the tracking error of (a).

利用模糊滑動模式前饋回授控制
進行壓電致動平台之精密控制

利用模糊滑動模式控制器配合前饋估測
式補償器對壓電致動平台進行追跡控制，由於模糊滑
動模式控制器與前饋補償器均不需事先知道系統之精
確數學模型，因此本研究所提出之控制器具有設計簡
單容易體現之優點。實驗結果顯示，利用本文所提出
之控制器，吾人可將1Hz、5Hz之弦波參考輸入追跡誤
差控制於輸入最大振幅之1.4%內，此外當輸入為10Hz
頻寬之隨機參考輸入時，前述之追跡誤差可控制於最
大振幅之4%內。

關鍵字: 模糊滑動回授控制, 模糊滑動前饋回授控制