Measurement of viscosity in a vertical falling ball viscometer

Ping Yuan*, Ben-Yuan Lin, Syu-Fang Liu

Department of Mechanical Engineering, Lee Ming Institute of Technology,

Abstract

This study proposes a measurement of dynamic viscosity by recording the falling time of a sphere ball from static to passing a certain distance. This method applies the Newton’s law of motion with the acceleration term and corrected Stokes’ law to derive the solution in terms of the viscosity and the falling time. Then, authors utilize a numerical program with an iteration scheme to calculate the viscosity, which is implicit in the solution. Through an experimental validation by the sample fluid of SAE 30, the results indicate that the error of measurement is below 6%, and the confidence interval is ± 0.0034 with a confidence level of 99.73%.

Keywords : Viscosity, Measurement, Falling Ball, Viscometer

1. Introduction

A number of methods are used to measure the viscosity of fluids, and they are typically based on one of three phenomena, which are a moving surface in contact with a fluid, an object moving through a fluid, and fluid flow through a resistive component. The three phenomena induce three major viscometers in the industry, which is rotating viscometer, falling-ball viscometer, and capillary viscometer, respectively. The falling ball viscometer typically measures the viscosity of Newtonian liquids and gases. This method applies the Newton’s law of motion under force balance on a falling sphere ball when it reaches a terminal velocity. In the Newton’s law of motion for a falling ball, there exists buoyancy force, weight force, and drag force, as well as these three forces reaches net force of zero. Meanwhile, the drag force can be got from the Stokes’ law, which is valid in Reynolds numbers less than one [1-3]. Falling ball viscometer is a well developed device for measuring viscosity of a fluid, and the method had claimed in international standards [4, 5]. In the international standards, the method is different to the principle described in [1-3]. The standards described an inclined-tube method, which the tube for ball falling was inclined at 10° to the vertical. Moreover, they used six balls with different diameter for various dynamic viscosity measurement ranges, and a suitable ball can be selected when the fall times of the ball is not lower than the recorded minimum fall times through a test procedure. Then, the rolling and sliding movements of a ball through the sample liquid are times in an inclined cylindrical measuring tube. The sample viscosity is correlated to the time, which a ball requires to drop down a definite distance, and the test results are given as dynamic viscosity. Brizard et al [6] developed an absolute falling ball viscometer, and made it possible to cover a wide range of viscosities while keeping a weak uncertainty. This method considered the effect of edge, inertial, etc., as well as corrected the measurement result to reach a relative uncertainty of the order of 10^{-3}. Capper Ltd. [7] claimed an improved instrument for the measurement of viscosity. This invention applied the first and second coils at different known positions around the falling tube to record the falling time in order to avoid the human error. Nevertheless, this apparatus is restricted in the application for a stainless steel ball.

Although the falling ball method has developed very well and claimed in the international standards, there are some inconvenient in the operation of this kind of viscometer. For example, the viscometer need six different diameter balls to measure different range of viscosity, and a user must run a test procedure for selecting a suitable ball. Moreover, it is difficult to make sure where the falling ball arrives the terminal velocity, i.e. how long the distance between the beginning record line and the starting fall position is enough or not. Additionally, the inclined tube viscometer has 10 degree to the vertical, so the falling ball is not just fall down but also has rolling down. This phenomenon is different to the conditions in the derivation of falling ball method [1-3]. Therefore, this study plans to develop a new method based on the traditional falling ball method, and derives a dynamic equation for describing a falling ball behavior in a vertical tube. Because the feature of this kind of viscometer is vertical to the ground, this study names it as the vertical falling ball viscometer.

2. Analysis

A falling ball viscometer is that a sphere ball falls down along a tube containing the sample liquid to be measured, and this tube is surrounded concentrically by
a tubular jacket for thermal control. The previous theory used the Newton’s law of motion for describing the falling ball reaching a terminal velocity, so the net force of gravity, buoyancy, and drag is zero.

\[ W - F_g - F_d = \left( \frac{\pi d^3}{6} \right) g - \left( \frac{\pi d^3}{6} \right) \gamma r - 3 \mu m_d d = 0 \]  

\[ (1) \]

Meanwhile, the drag force is expressed in the third term on the right side of the equal sign according to the Stoke’s law, which is valid in \( \text{Re} < 1 \). The Eqn. (1) can be easily expressed into the following form.

\[ \mu = \frac{\gamma_r - \gamma_s}{18u_t} \]  

\[ (2) \]

Meanwhile, \( \gamma \) is the specific weight, \( d \) is the diameter of sphere, \( u_t \) is the terminal velocity. The subscript \( s \) and \( f \) represent the sphere and fluid, respectively. The above equation can be simplified to the following form as same as claimed in the standard [4].

\[ \mu = K \cdot (\rho_s - \rho_f) \cdot l \cdot t, \text{ where } K = \frac{g \cdot d^2}{18l} \]  

\[ (3) \]

Here, \( l \) is the falling length, and \( t \) is the time passing the length of \( l \). When the material properties, geometry properties, and the falling time are known, the viscosity can be got from Eqn. (2) or Eqn. (3). In the standards [4, 5], the coefficient of \( K \) must be estimated by measuring a reference liquid with a known viscosity, because the ball is not pure falling. Then, the viscosity of a unknown liquid can be easily calculated in Eqn. (3), when the falling time is known.

This study focuses on the description about the motion of a sphere ball falling from the beginning to the end, so it exists the acceleration term in the Newton’s law of motion.

\[ W - F_g - F_d = m \cdot \frac{d^2}{dt^2} \]  

\[ (4) \]

Equation (4) can be expressed in the following after substituting the volume expression of a sphere.

\[ \rho_s - \rho_f \cdot g - \frac{3C_d}{4d} \cdot \rho_s \cdot u_t^2 = \frac{d^2}{dt^2} \]  

\[ (5) \]

The drag coefficient of a sphere, \( C_d \), has been experimented and plotted the results versus Reynolds number [8-10]. On the assumption that the inertia terms in the equation of motion of a viscous fluid can be neglected in comparison with the terms involving the viscosity, Stokes law states the drag coefficient as the form, \( C_d = 24 / \text{Re} \). Figure 1 depicts the drag coefficient versus Reynolds number with the experimental results and two curve fitting lines. The solid square, dashed line, and continuous line represents the experimental data, the line of Stokes expression, and the line of \( C_d = 30 / \text{Re} \), respectively. It is clear that the continuous line is much better than the dashed line based on the consideration of matching the solid square when the Reynolds number is less than five, so this study selects the relationship between the drag coefficient and the Reynolds number as the follow.

\[ C_d = \frac{30}{\text{Re}} \]  

\[ (6) \]

Then, this study substitutes Eqn. (6) into the Eqn. (5) and get the following form.

\[ y + by = c, \text{ where } b = \frac{45\mu}{2\text{d}^2\rho_s} \text{, and } c = \frac{\rho_s - \rho_f}{\rho_f} \cdot g \]  

\[ (7) \]

The initial conditions are

\[ y = 0, \text{ and } \frac{dy}{dt} = 0 \text{, when } t = 0 \]  

\[ (8) \]

The exact expression of Eqn. (7) can be solved as the following.

\[ y = c \cdot \frac{b}{s^2} e^{s \cdot x} + \frac{c \cdot b - c}{s^2} \]  

\[ (9) \]

Equation (9) is difficult to find an explicit expression for the viscosity through the known parameters and the falling time, because the viscosity is implicit in the equation. Therefore, this study designs a numerical program to pursue the viscosity in Eqn. (9) by the iteration scheme. The process of the numerical program is that 1). Set the parameters of material and geometry. 2). Calculate the distance of \( y \) from Eqn. (9) by guessing a viscosity. 3). Check the relative error between the calculating distance and the falling distance, and correct the guess value of viscosity. 4). Repeat the Step 2 and Step 3 until the relative error of distance is below the convergence criteria. 5). Check the Reynolds number whether it is less than five. If it is not satisfied, the result must be cancel and change another ball with different density or diameter.

### 3. Experiment

This study arranges the experimental devices as shown in Fig. 2(a), which includes the fall tube containing the liquid to be measured, a tubular jacket for thermal control, a water reservoir with a heater, a temperature control system, a pump for circulation, and a camera. This camera has four million pixels, and gets a picture per 1/30 sec. The temperature control system can control the water temperature deviation less than 0.5 \(^\circ\text{C}\) by a feedback from a sensor in the tubular jacket. Figure 2(b) is the photo of the whole device layout except the camera. In this figure, the device in the right hand is the water reservoir with a heater and a pump, which connects to the tubular jacket by soft pipes. The device in the middle of Fig. 2(b) is the power and control box, which supplies the power to the pump and heater in the reservoir and control the on/off of the heater to keep a setting temperature of water. The left device in this photo is the main structure for the falling ball. The fall tube and the tubular jacket can be inverted by rotating them together about their mounting point on the stand in order to return the ball to the starting position. On the top of the fall tube, there is a pin for fixing the ball against falling, and it can be released by the control of the control box. The stand is equipped with a spirit level and feet with leveling screws for adjusting the fall tube to be vertical to the ground.
The experimental process is that 1.) pour the liquid under test into the fall tube and puts the ball into the tube. Then, tight the screw on the top of the fall tube. 2.) Setting the measurement temperature and turn on the heater and pump. It needs at least 15 min for waiting the tubular jacket temperature arriving the specified test temperature. 3.) Rotate the fall tube 180°, and fix the pin for holding the falling ball. Then, rotate it back and ready to measure the falling time. 4.) Set up the camera to be perpendicular to the fall tube, and begin to record. 5.) Release the pin for dropping the ball, and stop the camera record until the fall ball passing through the marked end line on the fall tube. 6.) Record the falling time from the beginning to the end, and follows the calculating step in the above section to get the dynamic viscosity.

4. Results and Discussion

This study uses an oil of SAE 30 as the sample liquid for measurement. It is belong to the CD/SF level in the classification of API, and its kinetic viscosity is 111.2 cSt at 40 °C, which is equal to 0.0973 Pa·s of dynamic viscosity. This sample oil has a density of 894.4 kg/m³ through a measurement by an electrical balance with a decision of 1mg. Moreover, this study selects a plastic sphere ball as the falling ball, which has a diameter of 9.756mm, out of roundness of 0.09mm, and density of 1010 kg/m³. The experiment is proceeding under a 40 °C tubular jacket temperature, and four different falling distances of 5, 10, 15, and 20 cm. Furthermore, this paper repeats ten times for measuring fall time in each case, and the experimental results are listed in Table 1.

Figure 3 depicts the relative error of the measuring dynamic viscosity in Table 1 to the reference value of SAE 30 oil. The continuous, dashed, dash-dotted, and long dashed line represents the error in different falling distance of 5, 10, 15, and 20 cm, respectively. In this figure, the relative error rank in decent order globally is continuous, dashed, dash-dotted, and long-dashed line. That means the longer measuring distance can reduce the measuring error. In Fig. 3, all relative errors are between ±10%, and the average relative error in different falling distance is 3.9, 3.4, 2.9, and 2.2%. Therefore, this method of measuring dynamic viscosity is reliable.

This study refers the metrology in evaluating measurements of a single measurand and expression of uncertainty to analyze the uncertainty of this experiment [11]. Table 2 lists the mean of results, the experimental standard deviation, the uncertainty, and the confidence internal, which are defined in the follow.

\[
\mu = \frac{1}{n} \sum_{j=1}^{n} \mu_j \tag{10}
\]

\[
s = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (\mu_j - \mu)^2} \tag{11}
\]

\[
\mu = \bar{\mu} \pm 4.094 u \tag{13}
\]

In Table 2, the uncertainty and the confidence internal decreases with the increase in the falling distance, and the confidence internal is ±0.0034 with a confidence level of 99.73%. This represents that the error of measurement is about 6%, which is an acceptable experiment error.

In the experiment of this study, authors consider that there are two factors affecting the experimental results, which are the resolution of the camera and the curve fitting equation in the Fig. 2. When the pin for fixing the falling ball is released, it represents the ball beginning to fall. In this study, authors check the pictures recorded per 1/30 s one by one, and try to find one of them recording the starting moving as well as one of them recording the ball passing the end line. Nevertheless, it is little hard to distinguish which picture record the pin moves away the falling ball when the camera’s resolution is not good enough for zoom in the pin. Therefore, an excellent camera can promote the accuracy and stability of the measurement of dynamic viscosity in the vertical falling ball viscometer. On the other hand, this study selects C_50°30/Re instead of the Stoke’s law of C_e=24/Re, because the curve fitting line of C_50°30/Re is much better in Fig. 2. If one uses the curve fitting line of C_e=24/Re, the relative error will increase to be over 10%.

There is a restriction in this measuring method, which is the Reynolds number must be less than five. Therefore, the sphere falling ball must be changed when the falling Reynolds number is larger than five, because it violates the derivation of Eqn. (7) in this study. Nevertheless, this method in this study extends the suitable range from Re<1 to Re<5, i.e. enlarges the available range of a certain falling ball. This means that the method can decrease the chance of changing falling ball in the experiment. Besides, according to the theory of Stokes’ law, the figure of drag coefficient versus Reynolds number depicts the properties of a perfect sphere ball moving in a liquid. In this research, although authors measure the out of roundness of the sphere ball, we still not analyze the error due to the effect of the out of roundness. Authors hope that we will extend to investigate the effect of out of roundness on the measurement results in the future work, and validate the accuracy of this method by measuring more reference liquids.

5. Conclusions

This study developed a new method for measuring the dynamic viscosity based on the falling ball viscometer. The characteristics of this method are that the Newton’s law of motion includes the acceleration term, and the dynamic viscosity must be calculated by a numerical
code with an iteration scheme. Because the falling tube is vertical to the ground, which is different to the devices claimed in international standards, this study calls it the vertical falling ball viscometer. This study used the SAE 30 oil as the sample liquid, and the results indicate that the confidence interval of measurement is ±0.0034 with a confidence level of 99.73%, and the measuring error of viscosity is close to 6%. The future work hopes to increase the measuring accuracy and stability by promoting the camera with a higher resolution and consider the effect of the out of roundness of the falling ball on the measurement.

6. References


Table 1 Measuring and calculating results of SAE 30 at 40 °C in the experiment

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Time (s)</th>
<th>Re</th>
<th>μ</th>
<th>Time (s)</th>
<th>Re</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.23</td>
<td>3.3991</td>
<td>0.1044</td>
<td>2.40</td>
<td>3.5170</td>
<td>0.1034</td>
</tr>
<tr>
<td>2</td>
<td>1.17</td>
<td>3.7708</td>
<td>0.0989</td>
<td>2.27</td>
<td>3.9395</td>
<td>0.0976</td>
</tr>
<tr>
<td>3</td>
<td>1.20</td>
<td>3.5776</td>
<td>0.1016</td>
<td>2.33</td>
<td>3.7355</td>
<td>0.1003</td>
</tr>
<tr>
<td>4</td>
<td>1.17</td>
<td>3.7708</td>
<td>0.0989</td>
<td>2.27</td>
<td>3.9395</td>
<td>0.0976</td>
</tr>
<tr>
<td>5</td>
<td>1.13</td>
<td>4.0540</td>
<td>0.0952</td>
<td>2.23</td>
<td>4.0850</td>
<td>0.0958</td>
</tr>
<tr>
<td>6</td>
<td>1.23</td>
<td>3.3991</td>
<td>0.1044</td>
<td>2.40</td>
<td>3.5170</td>
<td>0.1034</td>
</tr>
<tr>
<td>7</td>
<td>1.17</td>
<td>3.7708</td>
<td>0.0989</td>
<td>2.30</td>
<td>3.8355</td>
<td>0.0998</td>
</tr>
<tr>
<td>8</td>
<td>1.20</td>
<td>3.5776</td>
<td>0.1016</td>
<td>2.37</td>
<td>3.6082</td>
<td>0.1020</td>
</tr>
<tr>
<td>9</td>
<td>1.20</td>
<td>3.5776</td>
<td>0.1016</td>
<td>2.37</td>
<td>3.6082</td>
<td>0.1020</td>
</tr>
<tr>
<td>10</td>
<td>1.20</td>
<td>3.5776</td>
<td>0.1016</td>
<td>2.37</td>
<td>3.6082</td>
<td>0.1020</td>
</tr>
</tbody>
</table>

Falling length (5cm) Falling length (10cm)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Time (s)</th>
<th>Re</th>
<th>μ</th>
<th>Time (s)</th>
<th>Re</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.57</td>
<td>3.7681</td>
<td>0.1001</td>
<td>4.67</td>
<td>3.6869</td>
<td>0.1014</td>
</tr>
<tr>
<td>2</td>
<td>3.40</td>
<td>3.9268</td>
<td>0.0980</td>
<td>4.47</td>
<td>4.0276</td>
<td>0.0969</td>
</tr>
<tr>
<td>3</td>
<td>3.43</td>
<td>3.8575</td>
<td>0.0989</td>
<td>4.47</td>
<td>4.0276</td>
<td>0.0969</td>
</tr>
<tr>
<td>4</td>
<td>3.37</td>
<td>3.9979</td>
<td>0.0971</td>
<td>4.43</td>
<td>4.1014</td>
<td>0.0961</td>
</tr>
<tr>
<td>5</td>
<td>3.37</td>
<td>3.9979</td>
<td>0.0971</td>
<td>4.40</td>
<td>4.1581</td>
<td>0.0954</td>
</tr>
<tr>
<td>6</td>
<td>3.60</td>
<td>3.4977</td>
<td>0.1039</td>
<td>4.57</td>
<td>3.8516</td>
<td>0.0991</td>
</tr>
<tr>
<td>7</td>
<td>3.47</td>
<td>3.7680</td>
<td>0.1001</td>
<td>4.50</td>
<td>3.9735</td>
<td>0.0976</td>
</tr>
<tr>
<td>8</td>
<td>3.57</td>
<td>3.7681</td>
<td>0.1001</td>
<td>4.60</td>
<td>3.8010</td>
<td>0.0998</td>
</tr>
<tr>
<td>9</td>
<td>3.57</td>
<td>3.7681</td>
<td>0.1001</td>
<td>4.67</td>
<td>3.6869</td>
<td>0.1014</td>
</tr>
<tr>
<td>10</td>
<td>3.63</td>
<td>3.4395</td>
<td>0.1048</td>
<td>4.70</td>
<td>3.6396</td>
<td>0.1020</td>
</tr>
</tbody>
</table>

Falling length (15cm) Falling length (20cm)

Table 2 Uncertainty and the confidence interval in this experiment

<table>
<thead>
<tr>
<th>Fall Distance</th>
<th>μ</th>
<th>s</th>
<th>u</th>
<th>± 4.09μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5cm</td>
<td>0.01007</td>
<td>0.28006E-4</td>
<td>0.88562E-3</td>
<td>± 0.00362</td>
</tr>
<tr>
<td>10cm</td>
<td>0.10030</td>
<td>0.26843E-4</td>
<td>0.84885E-3</td>
<td>± 0.00347</td>
</tr>
<tr>
<td>15cm</td>
<td>0.10002</td>
<td>0.25890E-4</td>
<td>0.81900E-3</td>
<td>± 0.00335</td>
</tr>
<tr>
<td>20cm</td>
<td>0.09866</td>
<td>0.24114E-4</td>
<td>0.76255E-3</td>
<td>± 0.00312</td>
</tr>
</tbody>
</table>

Fig. 1 Drag coefficient versus Reynolds number of a sphere moving in liquid.
垂直式落球黏度計之黏度量測方法
袁平、林本源、劉旭昉
黎明技術學院機械系

摘要

本篇論文提出一黏度量測方法，其乃藉由量測一落球由靜止到通過一固定距離之時間而推算出待測液體之動黏度係數。此方法利用包含加速度項之牛頓運動定律及修正之史托克定律推導出以動黏度係數及落下時間所表示之解析解。而後，利用疊代法之數值程式計算出隱含在解析解中之動黏度係數。經由SAE30號機油之實驗測試，結果顯示黏度量測誤差在6%以內，且在99.73%的信賴水準下其信賴區間為±0.0034。

關鍵字：黏度、測量、落球、黏度計

Fig. 2 Schematic and photo of the experimental devices layout

Fig. 3 Relative error of measuring dynamic viscosity to the reference value in different falling distance